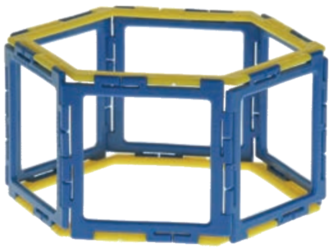
POLYDRON

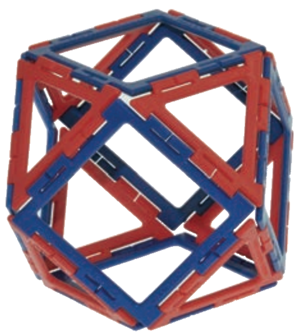
Teaching and Learning 3-D Geometry

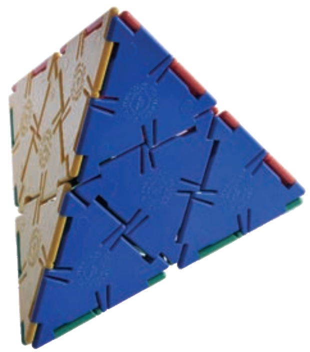
This publication is designed to support and enthuse primary trainees in Initial Teacher Training. It will provide them with the mathematical subject and pedagogic knowledge required to teach 3-D geometry with confidence.

The pack can also be used by practising teachers who wish to extend their own subject knowledge or who want to enrich children's opportunities and engage them with one of the most fascinating aspects of mathematics.



Polydron has been widely used in primary schools for over 30 years and has enriched the mathematical experiences of many thousands of children. Indeed, for many children Polydron has been their introduction to the wonderful world of 3-D geometry.





Using the Pack

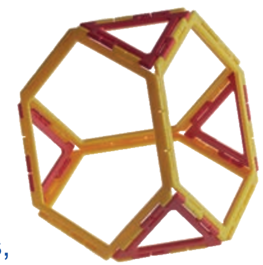
This pack provides sixteen pages of activities, each one supported by notes that offer students and teachers guidance on the subject knowledge required, effective pedagogy and what aspects of the National Curriculum are covered.

Also, the notes suggest probing questions so that you can determine the level of children's understanding and provide enrichment ideas to deepen children's knowledge.

The activities range across all aspects of the 3-D geometry curriculum related to plane shapes and polyhedra and many go beyond the requirements of the National Curriculum.

This gives teachers the opportunity to offer enrichment and to use the materials to develop problem-solving and spatial reasoning, in an atmosphere of enquiry, discussion and enthusiasm.

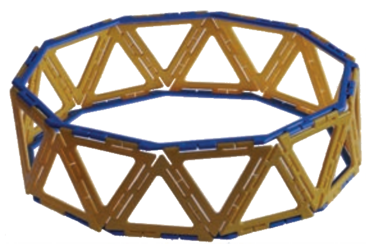
For example, the solid on the right appears quite straightforward. However, many children will struggle to assemble it even with a picture. Children need to develop the spatial reasoning skills that allow them to interpret a two dimensional photograph and to use this information to construct a 3-D model.



The solid shown possesses many interesting properties, since every vertex is identical and is composed of two hexagons and one equilateral triangle. This solid also has lots of symmetry to discuss and it is related closely to the tetrahedron, a solid that is in the National Curriculum.

Download this Pack

This pack can be downloaded from the free resources section of www.polydron.com



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Contents

The final four activities are extensions. While they are more demanding than the others, they are suitable for anyone with interest and confidence in using Polydron.

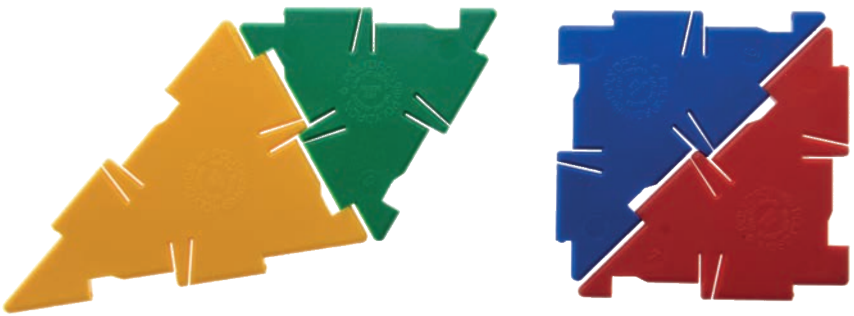
They will also be suitable to promote 3-D geometry amongst primary mathematics specialists or those aspiring to leadership in primary mathematics.

1

Working With Triangles –1

- There are four different triangles in Polydron.

- Explore different ways to join any two triangles together.
- You must join short edges to short edges and long edges to long edges.
- How many different pairs of triangles can you make? Here are two to start you off.



- How can you be sure that you have found them all?

Subject and Pedagogic Knowledge

This activity introduces children to the variety of triangles. There are four different ones; the small equilateral, the large equilateral, the isosceles and the right-angled.

Some care is needed since we want to focus on properties and not the names of triangles. With this in mind it can be seen that the right-angled triangle is also isosceles.

The enrichment activity below invites children to consider the names and properties of pairs of triangles joined together. It is possible to make a square, a rhombus, a kite and a parallelogram. Squares and rhombuses (rhombi) are also parallelograms because they have the properties of parallelograms. But when the term is used here, we are looking for a parallelogram that is neither a square nor a rhombus.

National Curriculum Links

Year 2 Pupils should be taught to identify and describe the properties of 2-D shapes, including the number of sides and line symmetry in a vertical line.

However, to allow children access to properties such as symmetry, you will need to use accurately drawn shapes on paper or card.

Probing Questions for Children

- How do you know that you have found all the different pairs that can be made? You may need to remind children that they must join short edges to short edges and long edges to long edges and that they can join two of the same triangle together.

Extension and Enrichment

Joining pairs of triangles will form quadrilaterals. Invite children to name and classify the quadrilaterals they have found. It is possible to make a rhombus, a kite, a square, a parallelogram (one that is neither a rhombus nor a square) and one or two that have no particular name.

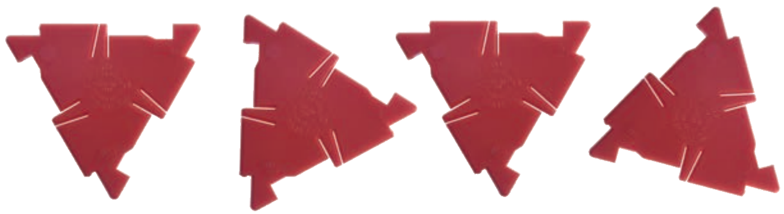
Conceptual Development

This activity supports children's conceptual development by focusing upon the properties of the shapes created and not simply being satisfied with naming them. In addition, children are introduced to the powerful ideas of finding all possibilities and with proving that you have them all.

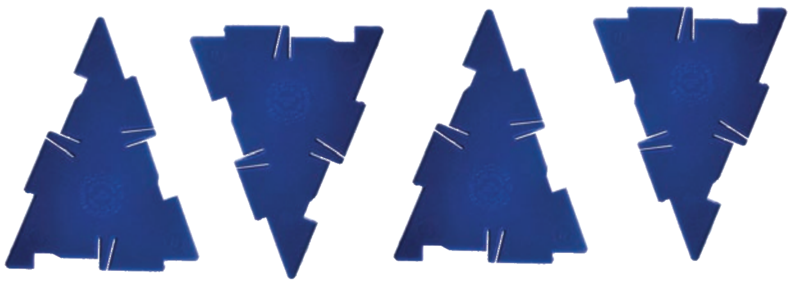
Working With Triangles – 2

- There are four different triangles in Polydron.

- Take four small equilateral triangles and create a larger equilateral triangle.



- Join four isosceles triangles together to make a large one that is similar to the small one.



- Join four right-angled triangles together to make a larger right-angled triangle.

Subject and Pedagogic Knowledge

This activity introduces children to the concept of enlargement by joining together triangles.

There are four different Polydron triangles; the small equilateral, the large equilateral, the isosceles and the right-angled.

Some care is needed since we want to focus on properties and not the names of triangles. With this in mind it can be seen that the right-angled triangle is also isosceles.

When creating a larger triangle it is important to use correct terminology. The larger triangles must be similar to the smaller ones. That is, the edges will be scaled by a factor of two. Many children find working with scale factor very demanding. You may find it helpful to use terms such as ‘linear scale factor’ and ‘area scale factor’ to clarify the difference.

National Curriculum Links

Year 6 Pupils should be taught to solve problems involving similar shapes where the scale factor is known or can be found.

Probing Questions for Children

Children will need to explain carefully how many triangles are needed for larger scale factors.

- Why will nine copies of any triangle make a larger one that is an enlargement scale factor 3?
- How many triangles are needed for an enlargement scale factor 4?

Extension and Enrichment

Making enlargements of triangles is not as straightforward as it looks. It takes care, insight and patience, especially if you invite them to enlarge a right-angled triangle.

Conceptual Development

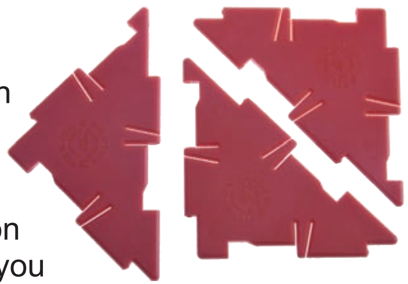
Many mathematical difficulties that children encounter later in school and beyond are based on a poor understanding of scale, in particular, on the relationship between linear, area and volume scale. Invite children to explain the relationship between one triangle and its enlargement using correct terminology. Avoid using terms such as ‘twice as big’, which are ambiguous.

3

Working With Triangles – 3

- There are four different triangles in Polydron.

- Use four small equilateral triangles to make this regular tetrahedron.
- Use four large equilateral triangles to make another regular tetrahedron.
- Use these four triangles to make an unusual tetrahedron.



- Here are three of the four triangles you need to make an unusual tetrahedron. Can you assemble it?
- Create a tetrahedron collection of as many different ones as you can.

Subject and Pedagogic Knowledge

This activity invites children to explore the tetrahedron, a solid made from four triangles. The plural of tetrahedron is tetrahedrons or tetrahedra.

There are four different Polydron triangles; the small equilateral, the large equilateral, the isosceles and the right-angled.

Since many children only recognise the regular tetrahedron, it is important to focus upon the properties. A tetrahedron is a closed solid made from any four triangles. A tetrahedron is also a triangular based pyramid. Using the triangles available, many different tetrahedra can be made.

Introduce the activity by discussing with children the two regular tetrahedra of different sizes and then explain that they are going to make others that are different from these. Explain that the word ‘regular’ when applied to a solid means that every face is made from regular polygons and every vertex is the same.

National Curriculum Links

Year 2 Pupils should be taught to identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces.

Probing Questions for Children

- Can you tell before you begin if the four triangles you have chosen might make a tetrahedron?

Extension and Enrichment

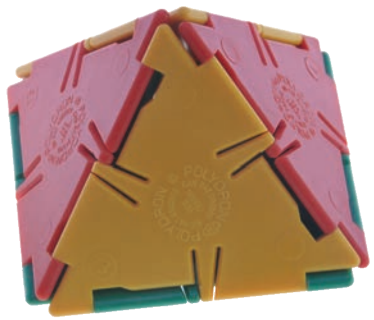
The final activity invites children to make a collection of all of the possible tetrahedra. Surprisingly, there are nine different ones that can be made with combinations of the four Polydron triangles (if you allow the two regular tetrahedra to be classed as different). The one shown on the right is the solution to one of the activities.

Conceptual Development

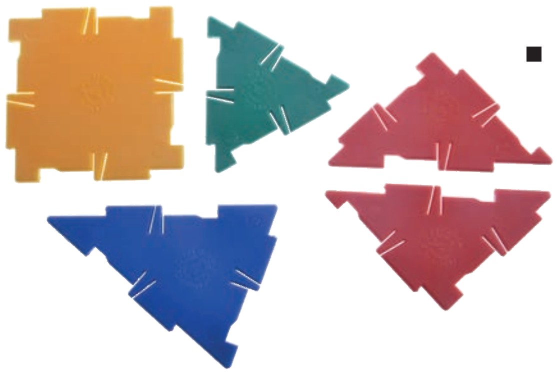
A critical aspect of children’s spatial development is the ability to discriminate between two 3-D solids that are in different orientations. This development can be encouraged with activities such as choosing two identical tetrahedra and one that is different. Place them in a variety of positions and invite children to find ‘the odd one out’ and explain why it is different.

4

Pyramids - 1

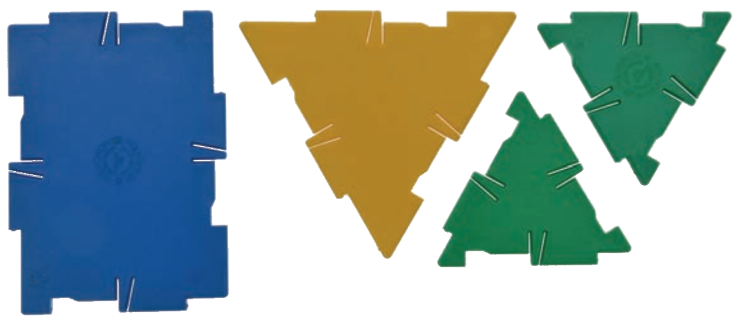


- Make this square-based pyramid. It has small equilateral triangles for the sloping faces.



- Make a square-based pyramid with these pieces.

- Use the pieces below to make a pyramid with a rectangle for a base. There is a piece missing from the picture.



- Make another rectangular-based pyramid with a different set of triangular sloping faces.

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Subject and Pedagogic Knowledge

A pyramid is a closed solid with sloping triangular faces and a base. While the traditional pyramid is square-based, children need to realise that any polygon can be used for the base and any triangles can be used for the sloping faces.

With Polydron it is possible to make a variety of pyramids. We can use any of the four sorts of triangle: the small equilateral, the large equilateral, the isosceles and the right-angled. We can also choose a variety of bases.

A standard pyramid is sometimes called a right pyramid. This is a pyramid in which the apex is above the centre of the base. When the apex is not above the centre of the base we can refer to it as a skew pyramid.

It may help your own knowledge to determine which of the pyramids in the activity is a right pyramid, and then go on to make two different rectangular based pyramids at the end, one that is a right pyramid and one that is skew.

National Curriculum Links

Year 1 Pupils should be taught to recognise and name common 2-D and 3-D shapes, including pyramids.

Year 2 Pupils should be taught to identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces.

Pupils should handle common 2-D and 3-D shapes and recognise these shapes in different orientations and sizes, and know that rectangles, triangles, cuboids and pyramids are not always similar to each other.

Probing Questions for Children

- What is the largest pyramid you can make with Polydron?

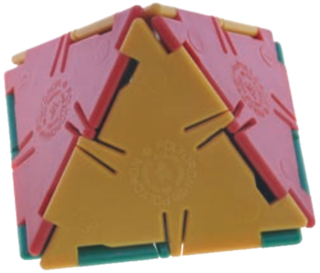
Extension and Enrichment

Can you make a pyramid from exactly 8 triangles? You may need to offer a hint. Four of the triangles are right-angled and are used to make the base. The other four are large equilateral triangles.

Conceptual Development

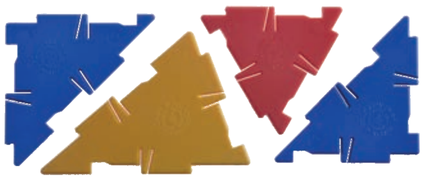
It is important for children's development of spatial reasoning that they see shapes in a variety of orientations and in different forms. In this activity, the intention is to allow children to generalise the properties of a pyramid by making ones that are increasingly different from the prototype they are used to.

Pyramids - 2



- This pyramid has a square base and four equilateral triangles for the sloping faces.
- Make one with each edge twice as long. You will need four squares for the base.

- Make this hexagonal-based pyramid.
- You need six sets of these pieces or Frameworks pieces of the same shape.



- Make each set into an identical unusual tetrahedron.
- Assemble your six tetrahedra to make a pyramid the same shape as the one above.

Subject and Pedagogic Knowledge

Complete 'Pyramids - 1' before trying this activity as it is important that children are familiar with the wide range of possibilities when working with pyramids.

With Polydron it is possible to make a variety of pyramids. We can use any of the four sorts of triangle: the small equilateral, the large equilateral, the isosceles and the right-angled. We can also choose a variety of bases.

The final activity shows how a pyramid can be made from smaller, triangular based pyramids or tetrahedra. The hexagonal pyramid in the activity uses Polydron Frameworks and the individual tetrahedra are photographed in Polydron. You can choose either, or mix them together.

National Curriculum Links

Year 1 Pupils should be taught to recognise and name common 2-D and 3-D shapes, including pyramids.

Year 2 Pupils should be taught to identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces.

Pupils should handle common 2-D and 3-D shapes and recognise these shapes in different orientations and sizes, and know that rectangles, triangles, cuboids and pyramids are not always similar to each other.

Extension and Enrichment

If children have completed the extension and enrichment activity in Pyramids – 1 then invite them to break the solid they have created into four identical, smaller solids. They will need to fill the gaps created with new pieces. The solution is shown on the right.

Conceptual Development

It is important for children's development of spatial reasoning that they see shapes in a variety of orientations and in different forms and that we avoid restricting their experience to the standard pyramid.

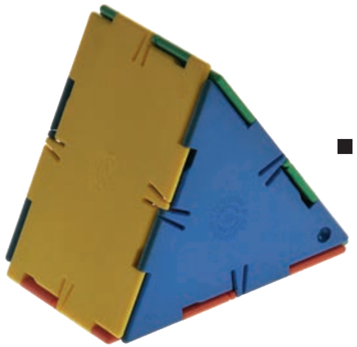
Language development can be supported by using terms such as rectangle-based to accurately describe a pyramid.

6

Prisms

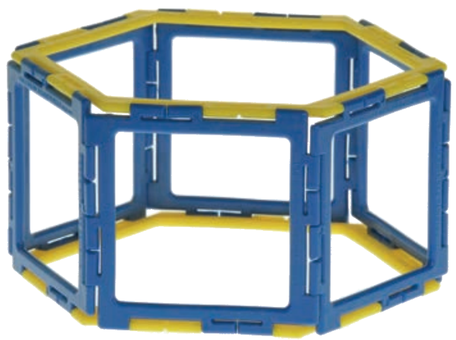
- Make this prism. You need three squares and two small equilateral triangles.

- Make the prism below. It has rectangular faces.



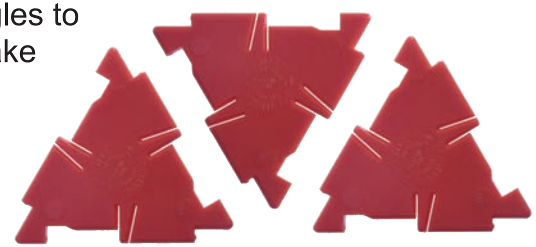
- Look at the two prisms together and decide what is the same about them and what is different.

- Make a prism with a hexagonal base.



- Join these three triangles to make a trapezium. Make another trapezium the same.

- Use them as the ends of an unusual prism.



Subject and Pedagogic Knowledge

With Polydron it is possible to make a variety of prisms. While a prism can be defined very precisely, a satisfactory definition for use in the primary classroom is a solid with two identical (congruent) polygons as end faces, surrounded by a belt of rectangles. Remember that squares are also rectangles.

A prism has an equal cross section along its length. This property sometimes leads teachers to identify a cylinder as a prism, but it is not one, as it has circular ends, and circles are not polygons. Polygons have straight edges.

Special prisms include cubes and cuboids, so you might like to begin a lesson on prisms by making these first.

National Curriculum Links

Year 1 Pupils should be taught to recognise and name common 2-D and 3-D shapes.

(The prism is not mentioned in the Year 1 curriculum by name but it is a straightforward enough solid to be included.)

Year 2 Pupils should be taught to identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces.

Pupils should handle and name a wide variety of common 3-D shapes including prisms.

Probing Questions for Children

- Can you make a prism with exactly 12 vertices?
- Can you make a prism with 9 vertices? If not, why not?

Extension and Enrichment

Enrich children’s experience with activity 10, ‘Antiprisms’. While antiprisms do not feature in the National Curriculum, children frequently make them. You can invite children to compare the properties of prisms and antiprisms.

Conceptual Development

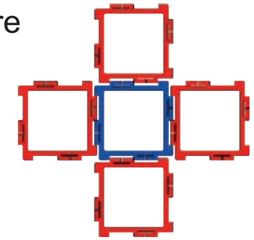
This activity strengthens children’s development of spatial reasoning by widening their experience of the concept of a ‘prism’. If they are restricted to the standard or prototype prism, with ends made from equilateral triangles, then this will restrict their access to more advanced concepts.

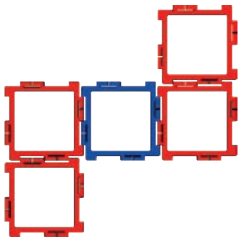
7

Open Cubes

■ Take four red squares and one blue square and make this net of an open cube. Notice how the squares are arranged.

■ Notice that the blue square is at the base or bottom of the open-topped cube.



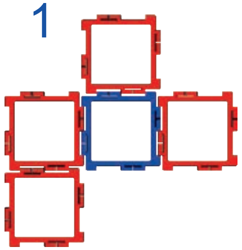


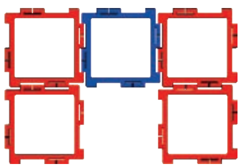
■ Now make this net. Decide if the blue square will be on the base of the cube before you fold it up.

■ Before you make the nets below, try to decide if each one will fold to make an open topped cube. Will the blue square be on the base of the open cube?

■ Make each one to check.

1





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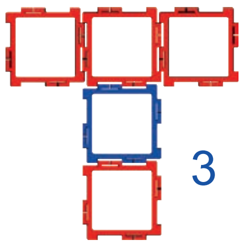


3

■ Make a complete set of nets for open-topped cubes.

Subject and Pedagogic Knowledge

This activity invites children to create, analyse and explore the nets of open cubes. This is a useful activity before embarking on the nets of a cube. There are fewer squares and fewer ways to arrange them.

The key mathematical interest in this topic is to encourage children's spatial reasoning by inviting them to visualise the folded shape before checking to see if it will fold to make an open cube.

National Curriculum Links

Surprisingly nets do not appear in the National Curriculum until Year 6.

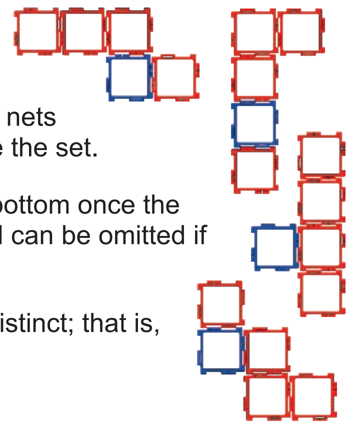
Year 6 Pupils should be taught to recognise, describe and build simple 3-D shapes, including making nets.

However, this activity is not only accessible to children much earlier, but it also encourages spatial reasoning and promotes mathematical discussion.

Probing Questions for Children

- How can you decide if the arrangement makes an open cube before you fold it up?
- How can you decide if the blue square will be the bottom of an open-topped cube before you fold it up?

Extension and Enrichment



The final question invites children to make a full set of nets for open-topped cubes. The four on the right complete the set.

They should arrange for the blue square to be at the bottom once the net is folded. This adds another level of challenge and can be omitted if necessary or if resources are limited.

Ask children how they can be sure that two nets are distinct; that is, different from each other?

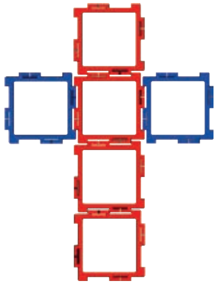
Conceptual Development

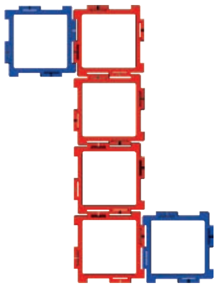
Nets combine between working in 2-D and working in 3-D. The manipulation of shapes from 2-D to 3-D, while a valuable experience, is not enough on its own to develop children's spatial reasoning. They need to be encouraged to work with images in their head and to combine this with manipulating the shapes.

8

Nets of a Cube

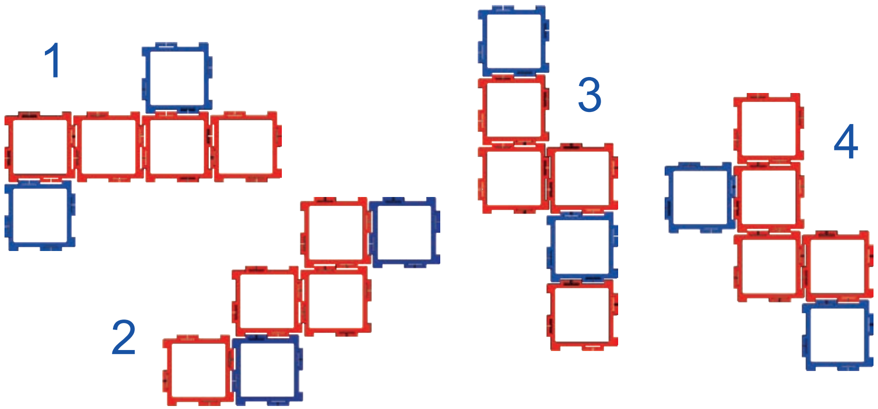
- Take four red squares and two blue squares and make this net of a cube. Notice how the squares are arranged.
- Check that the two blue squares are on opposite sides of the cube.





- Now make this net. Decide if the blue squares will be on opposite sides of the cube before you fold it up.

- Before you make the nets below, try to decide if each one will fold to make a cube. Will the two blue squares be on opposite sides of the cube?
- Make each one to check.



Subject and Pedagogic Knowledge

This activity invites children to create, analyse and explore the nets of a cube. The key mathematical interest in this topic is to extend children's spatial reasoning by inviting them to widen their experience of nets from the traditional cross shape. It also invites them to try to visualise the folded shape before checking to see if the six squares are nets.

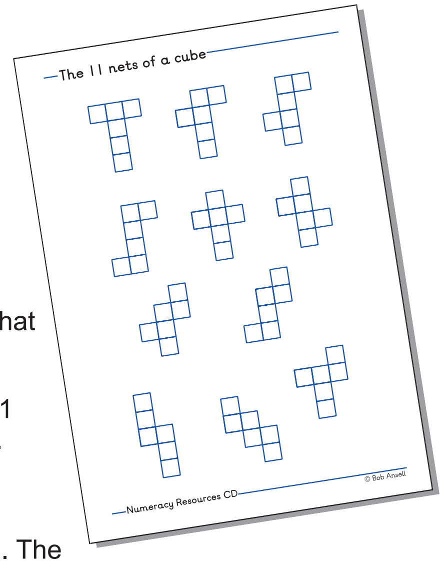
National Curriculum Links

Year 6 Pupils should be taught to recognise, describe and build simple 3-D shapes, including making nets.

Probing Questions for Children

- How can you decide if the arrangement makes a cube before you fold it up?
- How can you decide if the two blue squares are on opposite sides of the cube before you fold it up?

In fact all four of the arrangements at the bottom of the activity are nets of a cube. But some are harder to spot than others.



Extension and Enrichment

How many different nets can you find? Make each one and check. Draw each net on squared paper.

How can you be sure that two nets are distinct; that is, different from each other?

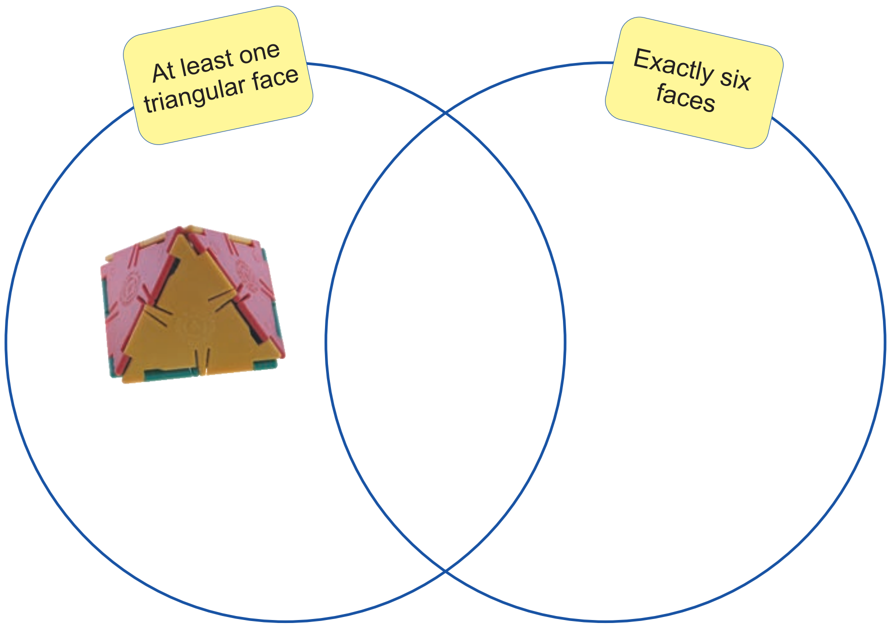
It could become a class challenge to create all 11 nets of a cube. The full set is shown on the right.

Conceptual Development

Nets combine working in 2-D and working in 3-D. The manipulation of shapes from 2-D to 3-D, while a valuable experience, is not enough on its own to develop children's spatial reasoning. They need to work with images in their head and to combine this with manipulation of shapes. They should be encouraged to make a conjecture based upon reasoning and then be allowed to test the conjecture in practice.

Working with Venn Diagrams

- You will need two loops and some post-it notes or small pieces of paper.
- Make a Venn Diagram like this.

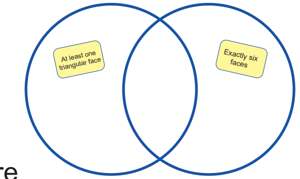


- Place solids into each of the four regions, including outside of both loops. One solid has been placed already.
- Choose some properties of your own and try to place a solid in each of the four regions, including the region outside.

Subject and Pedagogic Knowledge

This activity requires children to use mathematical language precisely and the Venn Diagram is a good method to encourage this. This activity works well with hoops and post-it notes on the floor.

You need to ensure that children label the edges of each loop as shown in the activity. Some children may be tempted to place the property label inside the set, as shown on the right. This can lead to confusion since it focuses attention on a portion of the set and tends to ignore the intersection.



Incorrect placement of the labels.

National Curriculum Links

Year 2 Pupils should be taught to identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces.

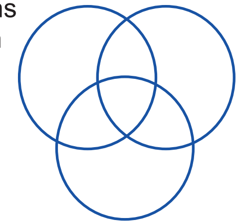
Pupils should identify, compare and sort shapes on the basis of their properties and use vocabulary precisely, such as sides, edges, vertices and faces.

Probing Questions for Children

- What will be the properties of a solid that is in the intersection of both sets? For those struggling, a pentagon-based pyramid is one option.
- What will be the properties of a solid that lies outside both sets?

Extension and Enrichment

Add a third loop as shown on the right. This creates eight regions including the one outside. The properties will need to be chosen carefully if you want a solid in each region.

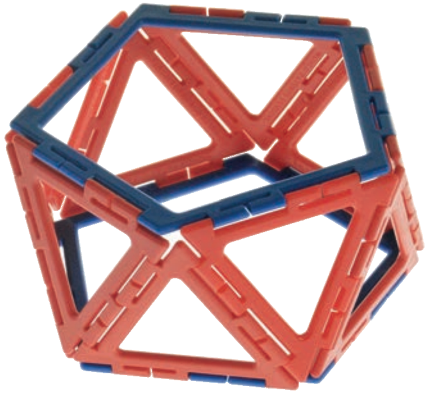


It could become a class challenge to come up with properties that achieve this. A photographic record of progress would help children.

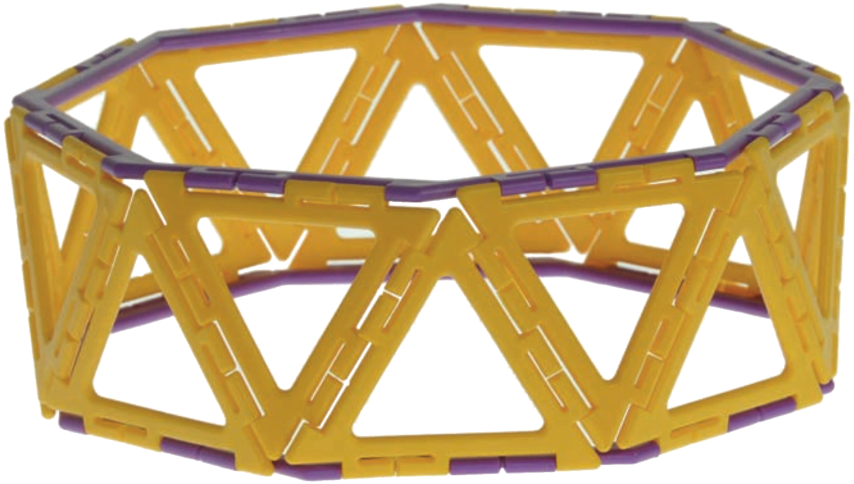
Conceptual Development

Developing children's ability to reason is a critical aspect of the mathematics curriculum. To support this development children need to use mathematical language in context correctly. This is a good opportunity to encourage children to use the correct terms of 'face', 'edge' and 'vertex' and to be precise when they articulate their reasons for placing each solid.

Antiprisms



- Make this antiprism.
- It is like a prism but has a belt of equilateral triangles instead of squares or rectangles.
- This is an octagonal antiprism.
- Work out which pieces you need before making it.



- Make the largest antiprism you can.
- Join two antiprisms together.

Subject and Pedagogic Knowledge

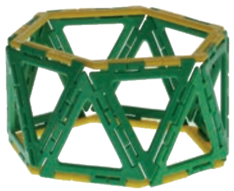
This activity requires children to create and describe unfamiliar solids. Antiprisms do not appear in the mathematics curriculum but if children are free to explore and create solids then one or two are very likely to be created. An antiprism is related to a prism. While a prism is a solid with two identical (congruent) polygons as end faces surrounded by a belt of rectangles, an antiprism is a solid with two identical (congruent) polygons as end faces surrounded by a belt of equilateral triangles.

National Curriculum Links

Year 2 Pupils should be taught to identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces.

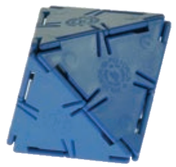
Pupils should identify, compare and sort shapes on the basis of their properties and use vocabulary precisely, such as sides, edges, vertices and faces.

Probing Questions for Children



- Can you work out how many triangles you will need for a hexagonal antiprism? You will need 12 triangles, with 6 attached to each of the two hexagons, as shown on the right.
- How do you join two identical antiprisms together?
- What is special about a triangular based antiprism?

Extension and Enrichment



A triangular antiprism is rather special as it will have an equilateral triangle for the base and top, and a belt of six equilateral triangles. These eight triangles form a regular octahedron, one of the five Platonic Solids, shown on the right.

Conceptual Development

We believe that it is important for children's development of spatial reasoning that they are allowed to create and work with unfamiliar solids, such as antiprisms.

The fact that these solids go beyond what is required by the National Curriculum does not stop children creating them and asking questions about them. As teachers, we need to encourage children's curiosity beyond the curriculum.

Platonic Solids - 1

- Here are three of the five solids named after the Greek philosopher, Plato.

The Tetrahedron

- This solid is made from four equilateral triangles.
- Three triangles meet at each corner or vertex.

The Cube

- The cube is made from six squares with three of them meeting at each vertex.

The Octahedron

- This octahedron is made with eight equilateral triangles.
- Four triangles meet at each vertex.

Subject and Pedagogic Knowledge

This activity is about three of the five Platonic Solids named after the Greek Philosopher, Plato. These solids are constructed from regular polygons and have vertex regularity, meaning that every vertex of a Platonic Solid is the same. The other Platonic solids are the regular dodecahedron and the icosahedron.

These solids are not named in the National Curriculum but many children will build them if they are given the opportunity to explore.

The names of the solids come from Greek words usually representing the number of faces.

National Curriculum Links

Year 2 Pupils should be taught to identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces.

Probing Questions for Children

- Record the number of faces, vertices and edges on each solid.
- How are you going to ensure that you do not miss any?

Extension and Enrichment

Euler's polyhedron formula states that for any convex solid made from polygons the number of faces, F, vertices, V and edges, E is connected by the formula $F + V - E = 2$.

Give children a variety of solids to explore and invite them to find the relationship for themselves. It makes sense to set the findings out in a table. Activity number 15 deals with this formula in more detail.

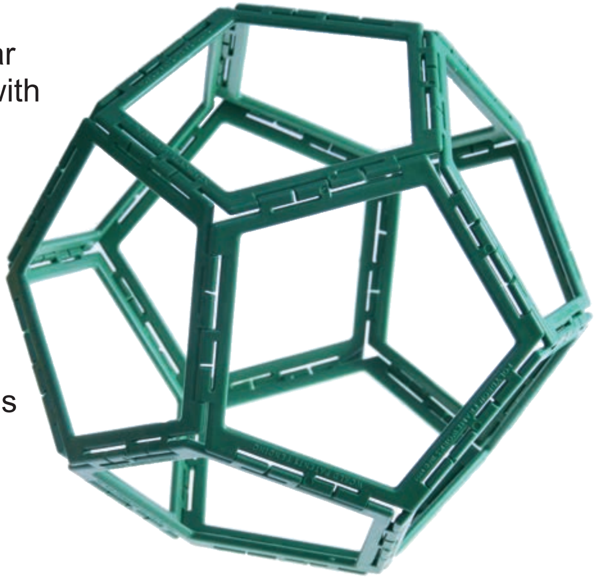
Conceptual Development

Go to the website nrich.maths.org and look up the van Hiele levels.

Which of the levels are being used when children make these solids?

What questions might you ask to develop their spatial reasoning towards the next level?

Platonic Solids – 2



- Make this regular dodecahedron with 12 pentagons.
- How many vertices does this solid have?
- How many edges does this solid have?
- This icosahedron needs 20 equilateral triangles.

- How many vertices does this solid have?
- How many edges does this solid have?
- Remove a group of five triangles and replace it with a pentagon.

See the Platonic Solids video at
polydron.co.uk/videos

POLYDRON

Subject and Pedagogic Knowledge

This activity is about two of the five Platonic Solids named after the Greek Philosopher, Plato. These solids are constructed from regular polygons and have vertex regularity, meaning that every vertex of a Platonic Solid is the same. The other Platonic solids are the regular tetrahedron, the cube and the regular octahedron.

These solids are not named in the National Curriculum but many children will build them if they are given the opportunity to explore.

Although it is more difficult to construct, some children may find it more satisfying to make the icosahedron with Polydron pieces as shown on the right.

National Curriculum Links

Year 2 Pupils should be taught to identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces.

Probing Questions for Children

- How are you going to count the vertices, faces and edges to ensure that you do not miss any?

Extension and Enrichment

Euler's polyhedron formula states that for any convex solid made from polygons the number of faces, F, vertices, V and edges, E is connected by the formula F + V - E = 2.

Give children a variety of solids to explore and invite them to find the relationship for themselves. It makes sense to set the findings out in a table. Activity number 15 deals with this formula in more detail.

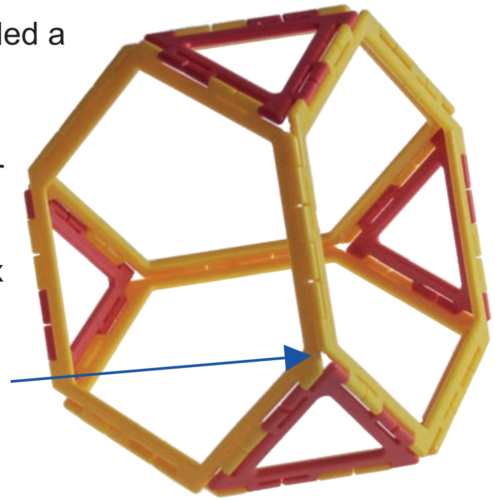
Conceptual Development

As with the conceptual development suggested in Activity 11, go to the website nrich.maths.org and look up the van Hiele levels.

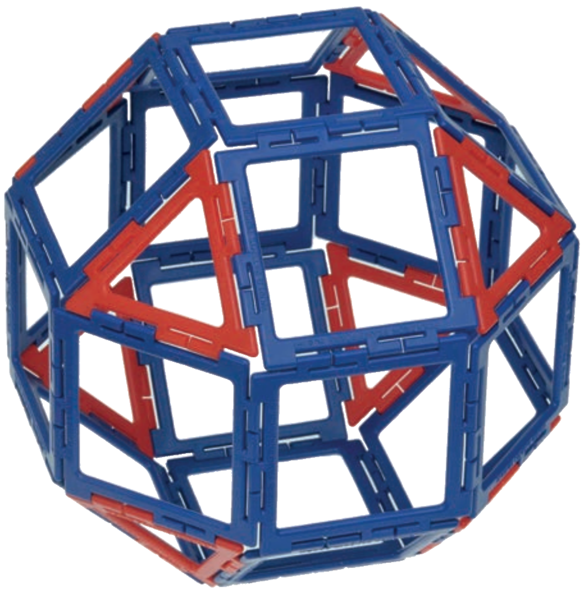
Now that children have experienced all five of the Platonic Solids, you can make use of the article on van Hiele levels to plan explicitly what is needed for them to make progress to a higher level. For example, children could be introduced to the concept of proof by trying to explain why there are exactly five solids with vertex regularity, each of which is constructed from just once sort of regular polygon.

Extension – Archimedean Solids 1

- Make this solid. It is called a truncated tetrahedron.
- As you construct it, focus on the corners, or vertices.
- Notice that every vertex is the same. At every vertex two hexagons and one triangle meet.



- This solid has the wonderful name of rhombicuboctahedron.



- Notice that every vertex is the same.
- At every vertex three squares and a triangle meet.

Subject and Pedagogic Knowledge

The Archimedean Solids are named after the Greek philosopher, mathematician and engineer, Archimedes. These solids do not appear in the National Curriculum but children love to build them. They are very satisfying, beautiful solids.

The first solid shown is the truncated tetrahedron. The word truncate means to cut or shorten. This solid is created by ‘cutting’ the corners from a tetrahedron.

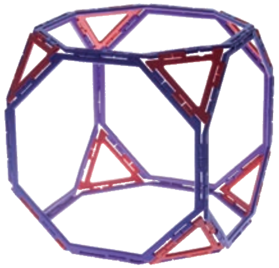
The key to creating any of the Archimedean Solids is to focus on the vertices. Every vertex is identical.

National Curriculum Links

There are no specific links or references to the Archimedean Solids in the National Curriculum. However, children enjoy the challenge of making them and the discipline and concentration required are attributes that we should be encouraging. In addition, the solids are very satisfying in their own right.

Probing Questions for Children

- Why do you think that 3.6.6 might be a shorthand way of describing the truncated tetrahedron?
- What would be the shorthand notation for the rhombicuboctahedron?
- Can you make a solid with vertex notation 3.8.8? This is the truncated cube shown right.



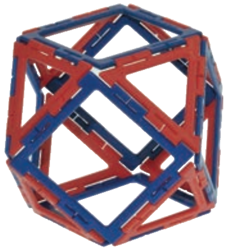
Extension and Enrichment

The truncated tetrahedron can be created from a tetrahedron, made with linear scale factor 3. That is, each face requires 9 small equilateral triangles. The truncation process then consists of removing a small tetrahedron from each vertex.

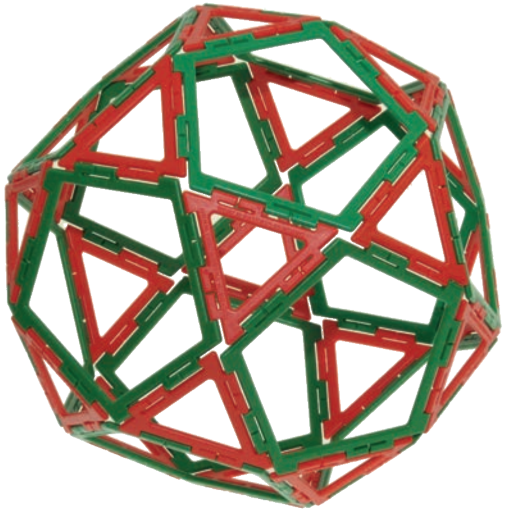
Conceptual Development

This is an opportunity for children (and teachers) to get to grips with a new notation. The notation commonly used describes the number and arrangement of the polygons that meet at a vertex. The truncated tetrahedron has notation 3.6.6 because one triangle and two hexagons meet at each vertex.

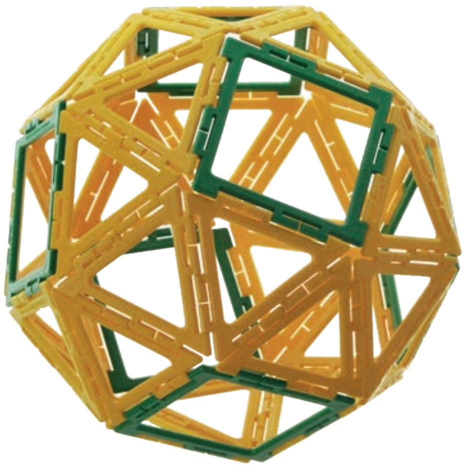
Extension – Archimedean Solids 2



- This solid is the cuboctahedron. It is made from 6 squares, like a cube and eight triangles, like an octahedron.
- Can you see why we say that every vertex is 3.4.3.4?



- For this solid you need 20 triangles and 12 pentagons.
- Each vertex has the arrangement 3.5.3.5
- It is called an icosidodecahedron. Can you work out where this name comes from?



- This wonderful solid is the snub cube.
- It is difficult to make as it has no symmetry to help you.

Subject and Pedagogic Knowledge

Complete Archimedean Solids - 1 before starting this activity.

This activity encourages children to focus their attention on the vertices of a solid and to create it using the vertex notation.

Most of the mistakes or difficulties that arise when creating Archimedean Solids stem from ignoring the vertex regularity of the solid.

The first two solids on the left can be created without too much difficulty. However, the third solid is demanding because it has no symmetry. We use symmetry extensively, often without realising it, to help us decide where to place the next polygon.

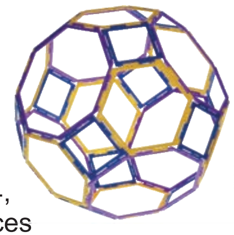
National Curriculum Links

There are no specific links or references to the Archimedean Solids in the National Curriculum. However, children enjoy the challenge of making them and the discipline and concentration required are attributes that we should be encouraging.

Probing Questions for Children

- What relationship can you see between the first two solids in the activity? What do you notice about their notations?
- How can you show that the snub cube has no symmetry?

Extension and Enrichment



Make a solid that has vertex arrangement 4.6.8. This solid is shown on the right, and is known as a truncated cuboctahedron.

For those children with very good visualisation skills invite them to find a relationship between the solid 4.6.8 and the solid 3.4.3.4, the cuboctahedron. The cuboctahedron has had each of its vertices removed with a plane cut, something you can demonstrate using elastic bands. Children will have to realise that there is also a significant change of scale involved.

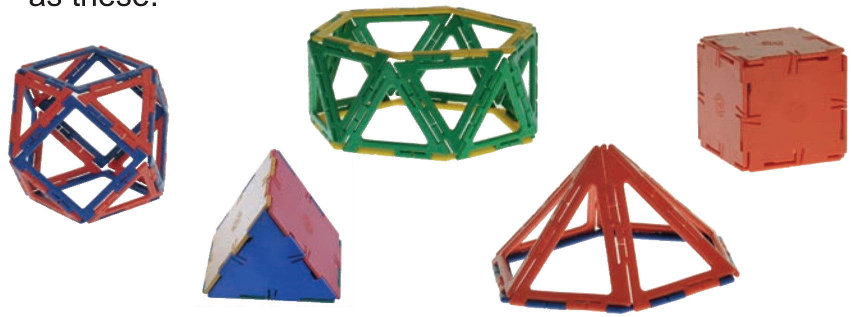
Conceptual Development

This activity promotes very high levels of visualisation and invites children to set themselves very demanding challenges.

15

Extension – Euler's Polyhedron Formula

- You need a collection of polyhedra for this activity, such as these.



- You are going to record the number of faces vertices and edges for each solid in a table like this.
- Extend the table with as many solids as you can find.

Name of Solid	Number of Faces F	Number of Edges E	Number of Vertices V		
Cube	6	12	8		

- Can you find a relationship between the number of faces, vertices and edges for each solid?
- Use the final two columns to explain your findings.

Enrichment – Euler's Polyhedron Formula

Subject and Pedagogic Knowledge

Euler's polyhedron formula states that for any convex solid made from polygons the number of faces, F, vertices, V and edges, E, is connected by the formula F + V - E = 2 . Sometimes this is written as F + V = E + 2.

With younger children, you might tackle the activity in a different way by giving them the formula and then invite them to verify it by examining a number of different solids. This would satisfy the National Curriculum requirement for Year 2, for example.

A conjectural approach is possible by inviting children to suggest a relationship. You can then invite them to see if their conjecture holds for another solid.

National Curriculum Links

Year 2 Pupils should be taught to identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces.

In addition, one of the three aims of the National Curriculum for mathematics is that children:

> can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions. (DfE 2013)

Probing Questions for Children

- How can you count the number of faces, vertices or edges reliably on a complex solid?

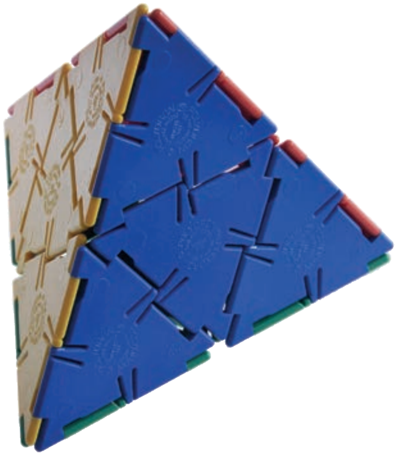
Counting faces, vertices and edges requires a systematic approach. It is usually much easier working with a partner than on your own.

Conceptual Development

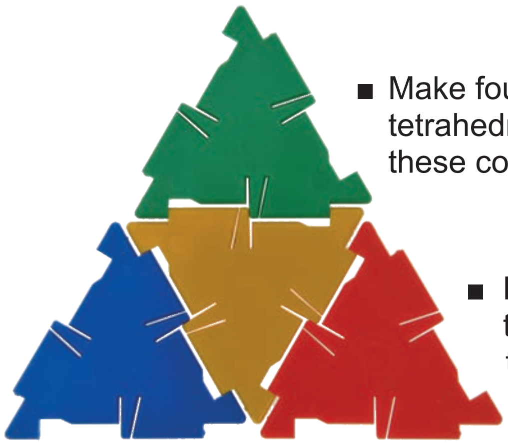
Euler's formula involves some confidence with algebra but as the relationship is relatively straightforward there is no reason why this activity could not be accessible to many children as a means to develop fluency with algebra.

It is important for later algebraic development that children realise that the letters F, V and E refer to the number of faces, vertices or edges and to avoid the temptation to use these variables rather like labels.

Extension – Tetrahedron Puzzle



- Work out from the picture how many small equilateral triangles are needed to make this solid.
- Make each face a different colour.
- Put your large tetrahedron to one side.



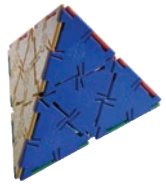
- Make four copies of a small tetrahedron using this net. Use these colours.
- Make an octahedron with two small equilateral triangles of each colour.
- Assemble all of your pieces to make the solid at the top of the page.
- Rearrange the pieces on the octahedron until the colours match the picture. Give your puzzle to a friend to solve.

Subject and Pedagogic Knowledge

Enlargement is a difficult mathematical topic for both primary and secondary children. In fact, most adults have difficulty with problems to do with scale.

This activity is designed to encourage children to explore informally the relationships between linear, area and volume scale factors.

This activity would be good for those teaching about such scale issues at all levels.



For example the tetrahedron shown in the puzzle has a linear scale factor of 2. This leads to each face having an area scale factor of $2^2 = 4$ and to a volume scale factor of $2^3 = 8$. Mathematics specialists may like to show that the volume of a pyramid is double that of a tetrahedron.

National Curriculum Links

One of the three aims of the National Curriculum for mathematics is that children:

> can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions. (DfE 2013)

This puzzle requires children to create the large tetrahedron by breaking it down into smaller pieces. Considerable thought and perseverance is required to assemble it and to arrange for the colours to match the original.

Probing Questions for Children

- How can you arrange for the colours to match the original?

Conceptual Development

Many children believe that you can assemble identical solids to make a larger copy of the solid. But this only holds true for cubes and cuboids.

However, it is good for them to see that a large tetrahedron can be created from familiar, smaller solids.